

This document constitutes the final report for NASA grant NAG5-4918 for the period spanning June 1999 -- Nov 2000, which includes a six-month no-cost extension. Questions or clarifications should be directed to the PI Keith Julien at (303) 492-5753 or [julien@colorado.edu](mailto:julien@colorado.edu).

The goal of this project was to theoretically and numerically characterize the waves generated beneath the solar convection zone by penetrative overshoot. 3D model simulations were designed to isolate the effects of rotation and shear. In order to overcome the numerically imposed limitations of finite Reynolds numbers ( $Re$ ) below solar values, series of simulations were designed to elucidate the Reynolds-number dependence (hoped to exhibit mathematically simple scaling on  $Re$ ) so that one could cautiously extrapolate to solar values.

We modified our parallel Chebyshev spectral solver to force individual and small numbers of cold plumes at the top of our numerical domain. We monitored the descent of individual plumes as they penetrated the underlying stable layer and generated gravity waves therein.

Analysis of the wave fields revealed the influence of lower boundary effects that rapidly dominated the numerical solutions. Further investigation demonstrated that the wave-radiative lower boundaries previously used by us and others (Klemp & Durran, 1983) when exploring convection-zone dynamics were unsatisfactory for diagnosing wave dynamics in the stable region. Hence, though the stable-layer boundary conditions do not adversely impact studies of the overlying convection-zone, their reflection characteristics overwhelm and dominate the dynamics in the stable layer below.

To address this difficulty, we examined alternate and complimentary approaches to the lower boundary condition, including:

1. Klemp and Durran hydrostatic, inviscid gravity-wave radiative conditions,
2. Klemp & Durran conditions with diffusion,
3. Klemp and Durran conditions with self-consistent high-order derivatives,
4. Rayleigh damping, and
5. Newton cooling.

All of our attempts with these and combinations of these boundary conditions produced unsatisfactory results. The most promising approach from these methods included a combination of 1., 4., and

5., with deep regions over which Rayleigh damping and Newton cooling slowly ramped from zero in the layer interior to peak values at the lower domain boundary. We learned while embarking on our investigation that the approach we found to work best is widely used in atmospheric meso-scale and global-scale modeling; it is often termed a "sponge-layer" technique. Despite its wide use, our results led us to conclude that this methodology is unsatisfactory because a) it requires too much of the numerical domain, and b) it does not damp large wavelength waves. In fact, once any simulation is run for longer than a wave-transit time across the layer, domain resonances were observed in which the entire stable layer is preferentially populated with the first few eigenmodes of the domain.

These unexpected results required a reorganization of our project. We concentrated our efforts in developing an improved formulation of gravity-wave transmitting boundary conditions so that we could proceed with our original plans (see project 2nd annual report). However, once initiating this unplanned effort, we anticipated that if we were to make progress in improving our understanding of rotating and sheared penetrative convection, we must undertake a more theoretically inclined approach until the time when our boundary-condition-development work was complete. Hence, our focus necessarily branched to two parallel courses of study: 1. improved wave-transmitting boundary conditions and 2. asymptotic reduced-PDE descriptions of rotating convection.

#### 1. Improved wave-transmitting boundary conditions.

In our course of research, we have reformulated all of the approaches to wave-radiating boundary conditions listed above as scattering problems to test their efficacy. We have analytically computed the reflection and transmission coefficients, and we have modified our Newton-Raphson-Kanterovich (NRK) solver to integrate the 1-D boundary-value problem of a monochromatic forced wave at the upper boundary and a specified lower boundary condition. The analytic reflection/transmission results are validated by the 1-D numerical work using NRK. The results demonstrate that each of the approaches enumerated above suffers from undesirable wave reflections from the lower boundary as well as internal wave reflections from the near-boundary damping region. Such unphysical reflections occur for all incident angles (the Klemp and Durran formulation performs as intended for normal incidence only). We judge the fundamental problem with all of these methods to be the inability to simultaneously increase the damping rate and reduce the amplitude of the

reflected wave; e.g., for both Rayleigh damping and Newton cooling, if one increases the damping coefficient for a smoothly varying damping profile, increased wave reflections result from the increasing gradient in the damping profile. One can reduce the gradient only by increasing the size of the numerical domain, devoting an increasing overall fraction of the domain to the unphysical damping layer. We consider this to be an unsatisfactory solution to the problem.

With the boundary-condition-formulation problem recast as a wave-scattering problem, improved techniques for removing unwanted wave reflections becomes clearer. The methods we have developed for addressing gravity-wave reflection is an integration of very recent developments in other fields, namely acoustic- and electromagnetic-wave propagation and reflection. The boundary-condition methods that have emerged in recent years in these fields have acquired the name "Perfectly Matched Layers" (or PML's). Several formulations exist in the literature. The most useful for our purposes include Hasthaven (1999) and Abarbanel et al. (1999) which examine acoustic waves in the Euler equations. Our approach follows closest the technique described in Abarbanel et al. (1999); however, because the problem investigated by Abarbanel et al. is fully parabolic, while ours includes an elliptic equation for the pressure, and furthermore because their work involves acoustic waves while we address gravity waves, significant differences appear.

The method we have developed (guided by Abarbanel et al, 1999) involves identifying the wave-dispersion relationship desired near the lower boundary, then adding appropriate wave-source and -sink terms to cancel (perfectly) any waves reflected from the interface between the domain interior and the near-boundary damping zone. The "desired" near-boundary dispersion relation in this case involves extreme damping of gravity waves propagating in any direction. By specifying wave-source and -sink terms that exhibit no reflection from the damping zone (i.e., that are perfectly matched with the layer interior), we can specify very large damping coefficients in thin damping regions with virtually no reflections exhibited. This allows a much smaller fraction of the numerical domain to be consumed by the unphysical damping layer. Whereas Newton cooling and Rayleigh damping may typically encompass as much as half or more of the numerical domain, with our approach the unphysical PML is confined to less than roughly 10 grid points near the boundary. We have demonstrated this with our 1-D NRK boundary-value integration code, and we are currently implementing the technique in our 3D Chebyshev spectral Boussinesq solver. Two manuscripts are currently in preparation.

We would like to stress that as the stably-stratified lower tachocline encompasses gravity, Alfvén, and inertial waves, the boundary-condition methodology we have developed offers new opportunities for detailed dynamical studies that were previously not possible.

## 2. Reduced Descriptions of rotating convection

As indicated in our first year report our research plan was modified slightly based on our derivation of a new class of reduced equations in the limit of rapid rotation. This limit is currently in the computationally prohibitive domain for current DNS simulations. These equations thus provide an asymptotically valid description for bulk motions of rapidly rotating flows, and in some sense represents the convective analogue of shallow water/QG slow manifold description for stable layer flows (ie they provide an asymptotic relaxation away from the Taylor Proudman constraint). However, we note that the present theory describes anisotropic coherent structures in the form of tall thin thermal plumes, as opposed to large aspect ratio structures.

Publications supported by this proposal on this particular aspect of the project are given below; some of the important results found include

- \* 3D simulations establishing that the reduced PDE's are indeed a slow manifold representation in qualitative agreement with computationally expensive Low Rossby number DNS simulations,
- \* extended applicability and bounds for Taylor-Proudman stiffening in rotation in strongly non-hydrostatic regimes,
- \* anisotropic diffusion of motions aligned with and perpendicular to the rotation axis, with strong asymptotically significant diffusion occurring in the latter, and
- \* exact single-mode analytic solutions to the reduced PDE's leading to planform-independent results that communicate the robustness of dynamics and statistics for general interior bulk flow (away from boundaries).

These results provide:

- 1) Establishment of near-boundary Ekman layers that passive couple the interior to the boundary so that boundary-layer matching can be rigorously employed, and more importantly omitted from any fully nonlinear reduced simulations;

- 2) A foundation for analytically decoupling the domains of boundary and interior flows crucial for the development of any parameterization scheme for such flows; and most importantly,
- 3) A potential better understanding of the stable/unstable layer coupling at the solar tachocline. With this in mind we have extend the slow manifold reduced equations to include magnetic fields.

In summary unanticipated difficulties with state-of-the-art treatment of the lower boundary conditions in stably stratified simulations prohibited implementation of our original work plan. However, we have developed new gravity-wave radiative conditions whose performance far surpasses the current state of the field by utilizing developments in other fields (e.g, acoustics and electromagnetic wave dynamics) that occurred subsequent to the initiation of our NAG5-4918 work. We expect the new method we have developed will be widely used by researchers both within and outside the astrophysical and solar communities, as open-boundary computational modeling presents a number of applications requiring numerically efficient wave-radiating conditions. We are eager to continue our originally proposed studies now that we have developed the required numerical techniques to diagnose uncompromised wave characteristics generated by overshoot dynamics.

Furthermore, the reduced-PDE descriptions we have developed for rotationally and magnetically constrained convection serve to theoretically guide future modeling work in these fields. We hope this ongoing work will make valuable contributions to our understanding of tachocline dynamics and processes.

#### Educational Outreach Supplement.

Our project includes an educational outreach supplement to develop Internet pages for K-12 classroom and independent learning. The intent of the educational component is to introduce students to solar scientists and to educate students on how computers are used to increase our understanding of solar processes and the solar interior. The educational outreach project is an on-going effort supported also by NASA contract NASW-99026.

Our strategy for the web pages includes three parallel components: 1) basic educational information on solar-physics, 2) a historical timeline outlining discovery related to solar physics, and 3) the

role computers have played in aiding discovery. Our goals are to produce a fun, interactive educational site that encourages exploration and identifies current and past researchers in solar physics. Thumbnail photos of practicing or past researchers will appear throughout the pages so students may explore particular branches of solar research. Rather than uninteresting headshots of scientists, we plan for the photos to indicate other interests (i.e., on a motorcycle, kicking a soccer ball, etc). In this way we hope to offer school children exposure to "cool" scientist role models so that the seeds of career-choice contemplation may be planted.

The current stage of development for the web pages includes a basic front-page design resembling the cockpit view from a space craft named "Solar Explorer". Controls allow the student to navigate through the solar wind, corona, photosphere, convection zone, tachocline, radiative interior, and core. Each page includes information about the sun as well as basic physics relevant to solar processes. At present we have accumulated information temporarily stored in individual html files. The content of the pages will be modified to target a 12-year old's vocabulary and comprehension and to integrate successfully with the rest of the web site. Current individual pages include: atom.html, cme.html, convection\_zone.html, core.html, doppler\_effect.html, dynamo.html, eandm.html, eclipse.html, energy.html, farside.html, flare.html, fusion.html, galileo.html, magnetism.html, mass.html, maunder.html, neutrino.html, nmsea.html, plasma.html, plasmaj.html, snp\_37cl.html, snp\_ray\_john.html, solar\_observing.html, solar\_waves\_main.html, solarcycle.html, source.html, sunspot.html, template.html, turbulence.html, vibration\_modes.html, viscosity.html. We hope the individual names provide some information about the content completed so far.

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Strongly Nonlinear Convection Cells in a Rapidly Rotating  
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Julien, K. and E. Knobloch,  
Journal of Fluid Mechanics 1998 360 141-178

A new class of equation for rotationally constrained flows  
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Journal of Theoretical and Computational Fluid Dynamics  
1998 11 251-261

Fully Nonlinear Three-Dimensional Convection in a Rapidly

Rotating Layer  
Julien, K. and E. Knobloch,  
Physics of Fluids 1999 11 1469-1483

Strongly Nonlinear Magnetoconvection in Three-Dimensions  
Julien K., E. Knobloch and S.M. Tobias  
Physica D 1999 128 105-129

Nonlinear Magnetoconvection in the Presence of Strong Oblique Fields  
Julien K., E. Knobloch and S.M. Tobias,  
Journal of Fluid Mechanics 2000 410 285-322

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M. Nunez and A. Ferriz-Mas Eds.,  
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Highly Supercritical Convection in Strong Magnetic Fields  
Julien, K., E. Knobloch and S.M. Tobias,  
Chapter 8 in Advances in Nonlinear Dynamos,  
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J. Comput. Physics 1999 142 129-147
- An upper boundary condition permitting internal gravity-wave  
radiation in numerical mesoscale models.  
Klemp, J.B. and Durran, D.R.  
Monthly Weather Review 1985 111 430-444.